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Shape optimization of arches and shells: an R-Funicularity based approach

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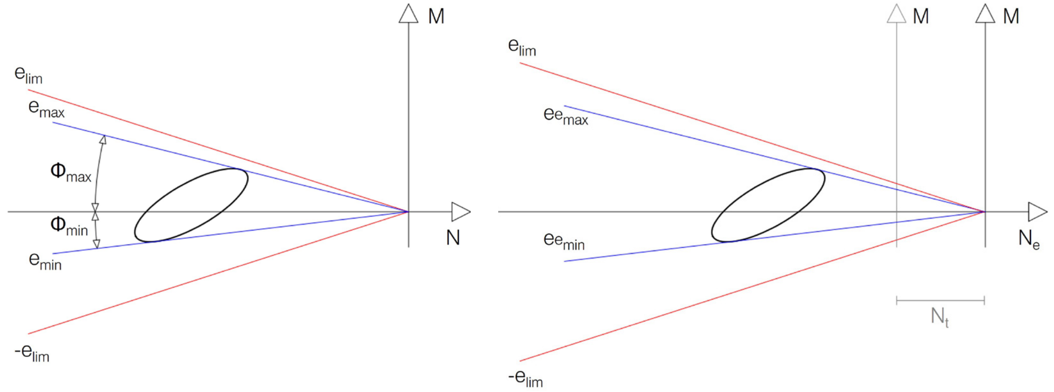
Abstract

Arches and shells, built and studied since the ancient times, are still very current today thanks to their efficiency when it is required to cover large spans. They are able, in fact, by means of a proper design of their curvatures, to behave in a funicular regime, i.e. to resist loads without introducing bending moments. Nevertheless, this mechanical response is possible under certain boundary conditions, load case and bending stiffness. Otherwise, bending arises and it is useful to measure how far the actual structural behaviour is from the funicular one by means of the eccentricity – generalized eccentricity for 2D structures – parameter. This measure has been used in 2018 to define the Relaxed Funicularity (R-Funicularity), a concept according to which a shell can be still considered funicular when the eccentricity belongs to an admissibility domain.

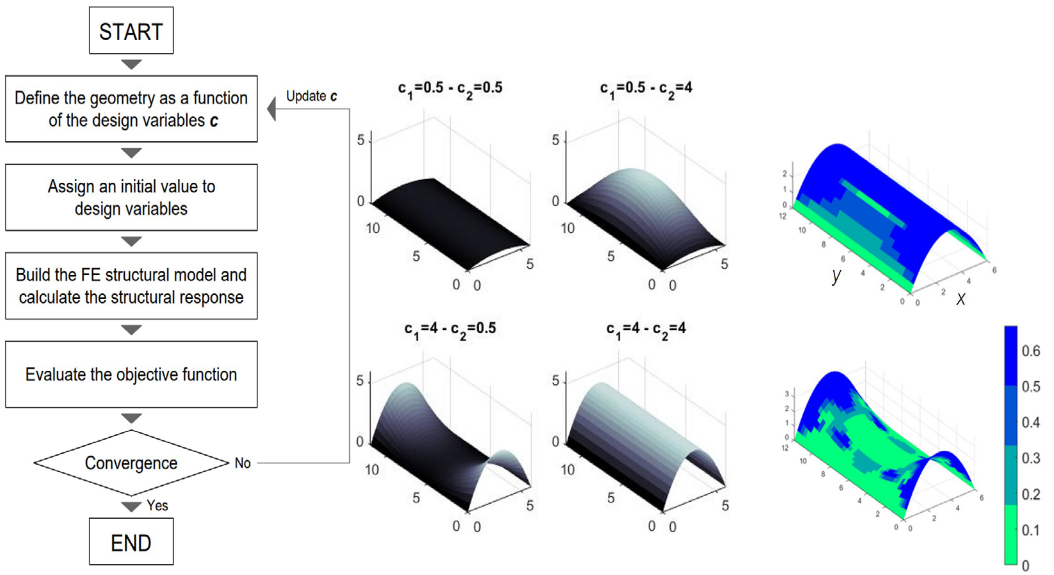
Basing on these premises, this dissertation exploits the R-Funicularity concept by proposing a new shape optimization approach aimed at finding R-Funicular arches and shells. This purpose is achieved by minimizing objective functions somehow depending on the generalized eccentricity. The advantages of shape optimization and the efficiency of funicular structures come together in a design tool whose main goal is to find an R-Funicular structure by manipulating its shape while accounting for its mechanical properties, boundary conditions and load case. The proposed shape optimization algorithm has been implemented in a new MATLAB based tool named *R-Fun Optimization*, integrated with the Finite Element analysis solver SAP2000.

R-Fun Optimization can be used to optimize the shape of both 1D and 2D structures. The geometry can be described by means of analytical or parametric functions, i.e. B-spline curves and surfaces. Numerical examples where non R-Funicular structures turn fully or mostly R-Funicular after the optimization process are proposed.

In addition, the R-Funicularity theory was extended by introducing the *eccentricity angle*, a parameter used to solve computational issues, and by proposing a formulation to consider the tensile strength in the admissibility domain. In this latter case, a parameter called *effective eccentricity* was introduced and used to span the extended domain. The obtained results are promising and suggest that an R-Funicularity based approach to shape optimization of arches and shells deserves to be further refined.



Left: Ellipse of eccentricity or RF ellipse. The generalized eccentricity extrema (e_{max}, e_{min}) are equal to the slopes of the blue lines. The admissibility domain ranges between e_{lim} and $-e_{lim}$, which are the slopes of the red lines. $\Phi_{max} = arctan\,(e_{max})$ and $\Phi_{min} = arctan\,(e_{min})$ are the eccentricity angles.
Right: Effective eccentricity extrema $(e_{e_{max}}, e_{e_{min}})$ and the extended admissibility domain when the tensile limit force N_t is taken into account.



Left: Main steps of the R-Funicularity based shape optimization process.
Centre: An example of design space obtained by modifying the boundary rises and the midpoint rise, respectively c_1 and c_2, of a Bi-Parabolic surface.
Right: Shells' shape and eccentricity distribution before (top) and after (bottom) optimization of a Bi-Parabolic concrete roof pinned along the y edge. The colorbar represents the maximum generalized eccentricity extrema that increases going from green to blue, the latter meaning that the eccentricity is out of the cross section. The shell is R-Funicular in the light green areas.